

SPIN & WIN:
MATHEMATICAL ANALYSIS & HOUSE ADVANTAGE

Prepared by

Robert C. Hannum, Ph.D.
Department of Statistics & Operations Technology
University of Denver
2101 S. University Blvd.
Denver, Colorado 80208
303-871-2115
rhannum@du.edu

May 18, 2004

Spin & Win: Mathematical Analysis and House Advantage

Introduction

Spin & Win is a casino game developed by Casino Creations and currently owned by Dragons Tail Enterprises. A mathematical analysis of a previous version of this game was prepared by Stanley Ko in May of 1999.¹ This present report presents a complete analysis of the current version of the game, including two new bets, the Single Joker and Double Joker, and provides explicitly the probability distribution of all possible final outcomes and overall game advantage, computed as the weighted average of the individual advantages for each of the possible bets, for various betting pattern scenarios. Relevant results are summarized in the tables in the Appendix: Tables 1 and 2 show final outcome probabilities, Table 3 lists house advantages for individual bets, Table 4 provides the overall game advantage figures, and Table 5 gives hit frequencies.

The Game and Payoffs

Spin & Win is played with an 8-sided spinner whose eight sides consist of 2 Red, 2 Green, 2 Blue, 1 Yellow, and 1 Joker. Players bet on the outcome of two spins of this spinner. There are twelve possible bets, broken down into three major types: Specific Color, Color Combo, and Joker bets. The player can make any of these bets. The Specific Color is a bet that a chosen color will appear at least once in the two spins. The Color Combo is a bet that chosen combination of two colors will appear, in any order, on the two spins. For the Specific Color and Color Combo bets, the Joker is wild and can substitute for any color. There are two Joker bets: Single Joker is a bet that the Joker will appear at least once in the two spins; Double Joker is a bet that the joker will appear on both spins. The following table gives winning payoffs on the different wagers:

SPIN & WIN PAYOFFS		
<i>Specific Color Bet</i>	<i>Payoff 1*</i>	<i>Payoff 2*</i>
Red, Green, or Blue	½ to 1	1 to 1
Yellow	1 to 1	2 to 1
<i>Color Combo Bet</i>	<i>Payoff</i>	
Red-Green, Red-Blue, or Green-Blue	2½ to 1	
Yellow-Red, Yellow-Green, or Yellow-Blue	4½ to 1	
<i>Joker Bets</i>	<i>Payoff 1*</i>	<i>Payoff 2*</i>
Single Joker	3 to 1	6 to 1
Double Joker	---	60 to 1
* Payoff 1 results if the chosen color/Joker appears only once in the two spins, Payoff 2 results if the chosen color/Joker appears on both spins.		

¹ Stanley Ko, "A Mathematical Analysis of *Spin to Win*," May 20, 1999.

Thus, if a player bets Blue (a Specific Color bet), he will be paid even money if the two spins are Blue and Blue, Blue and Joker, Joker and Blue, or Joker and Joker; he will be paid half his wager if the outcome is Blue-Red, Blue-Green, Blue-Yellow, Red-Blue, Green-Blue, Yellow-Blue, Joker-Red, Joker-Green, Joker-Yellow, Red-Joker, Green-Joker, or Yellow-Joker. A player betting on the Color Combo Yellow-Blue will be paid 4½ to 1 if the two spins are Yellow-Blue, Blue-Yellow, Yellow-Joker, Joker-Yellow, Blue-Joker, Joker-Blue, or Joker-Joker. A bet on Single Joker will be paid 3 to 1 if the Joker appears on only one spin and 6 to 1 if the Joker appears on both spins. The Double Joker bet pays 60 to 1 only if the Joker appears on both spins.

Notation

The letters R, G, B, Y, and J will represent the outcomes Red, Green, Blue, Yellow, and Joker respectively. Subscripts will be used to designate the outcome of a specific spin, first or second. Thus, R_1 is the event that the outcome of the first spin is Red, while Y_2 refers to the event that the second spin is Yellow. The event R_1Y_2 refers to the final outcome that the first spin is Red and the second spin is Yellow. Color Combo bets, where the order of occurrence of the two selected colors is unimportant, will be referred to merely in terms of the two colors involved, with no regard to order. Thus, Yellow-Blue would represent the same bet as Blue-Yellow.

Final Outcome Probabilities

If order is taken into account there are 25 possible outcomes from the two spins. These final outcomes and their probabilities are given in Table 1 of the Appendix. With respect to betting, however, the order in which colors appear on the two spins is irrelevant. When order is ignored, the number of possible outcomes reduces to 15, shown in Table 2 of the Appendix. Final outcome probability calculations in these two tables are based on the independence of the outcomes of the two spins and the following individual probabilities:

$$P(R_1) = P(G_1) = P(B_1) = P(R_2) = P(G_2) = P(B_2) = 2/8 = 0.25,$$

and $P(Y_1) = P(J_1) = P(Y_2) = P(J_2) = 1/8 = 0.125.$

The three examples below illustrate how final outcome probabilities are obtained:

$$P(\text{Red on 1}^{\text{st}} \text{ spin and Red on 2}^{\text{nd}} \text{ spin}) = P(R_1R_2) = 0.25 \times 0.25 = 0.0625.$$

$$P(\text{Yellow on 1}^{\text{st}} \text{ spin and Blue on 2}^{\text{nd}} \text{ spin}) = P(Y_1B_2) = 0.125 \times 0.25 = 0.03125.$$

$$P(\text{Yellow and Blue, any order}) = P(Y_1B_2 \text{ or } B_1Y_2) = 0.03125 + 0.03125 = 0.0625.$$

Other probabilities in Tables 1 and 2 are computed similarly.

House Advantage – Individual Bets

The house advantages for each of the 12 possible bets are shown in Table 3 of the Appendix. To show how these advantages are derived, the calculations for a Specific Color bet on Red, a Color

Combo bet on Green-Blue, and the two Joker bets – Single Joker and Double Joker – are shown below.

Specific Color Bet on Red:

$$\begin{aligned}
 P(\text{Red appears once}) &= P[(R_1G_2 \text{ or } G_1R_2) \text{ or } (R_1B_2 \text{ or } B_1R_2) \text{ or } (R_1Y_2 \text{ or } Y_1R_2) \\
 &\quad \text{or } (J_1G_2 \text{ or } G_1J_2) \text{ or } (J_1B_2 \text{ or } B_1J_2) \text{ or } (J_1Y_2 \text{ or } Y_1J_2)] \\
 &= 0.1250 + 0.1250 + 0.0625 + 0.0625 + 0.0625 + 0.03125 \\
 &= 0.46875.
 \end{aligned}$$

$$\begin{aligned}
 P(\text{Red appears twice}) &= P[(R_1R_2) \text{ or } (R_1J_2 \text{ or } J_1R_2) \text{ or } (J_1J_2)] \\
 &= 0.0625 + 0.0625 + 0.015625 \\
 &= 0.140625.
 \end{aligned}$$

$$\begin{aligned}
 E.V. &= (+0.5)(0.46875) + (+1)(0.140625) + (-1)(1 - 0.46875 - 0.140625) \\
 &= -0.015625.
 \end{aligned}$$

This represents a 1.5625% house advantage on the bet on Red. Similar calculations show the same advantage for Specific Color bets on Green and Blue, and a 6.25% edge for the Specific Color bet on Yellow.

Color Combo Bet on Green-Blue

$$\begin{aligned}
 P(\text{Green-Blue}) &= P[(G_1B_2 \text{ or } B_1G_2) \text{ or } (G_1J_2 \text{ or } J_1G_2) \text{ or } (B_1J_2 \text{ or } J_1B_2) \text{ or } (J_1J_2)] \\
 &= 0.1250 + 0.0625 + 0.0625 + 0.015625 \\
 &= 0.265625.
 \end{aligned}$$

$$\begin{aligned}
 E.V. &= (+2.5)(0.265625) + (-1)(1 - 0.265625) \\
 &= -0.0703125.
 \end{aligned}$$

This 7.03125% casino advantage on Green-Blue also holds for Color Combo bets on Red-Green and Red-Blue. The house advantage on any one of the Color Combo bets involving Yellow (i.e., Yellow-Red, Yellow-Green, and Yellow-Blue) is 5.46875%.

Single Joker Bet

$$\begin{aligned}
 P(\text{Joker appears once}) &= P[(J_1R_2 \text{ or } R_1J_2) \text{ or } (J_1G_2 \text{ or } G_1J_2) \\
 &\quad \text{or } (J_1B_2 \text{ or } B_1J_2) \text{ or } (J_1Y_2 \text{ or } Y_1J_2)] \\
 &= 0.0625 + 0.0625 + 0.0625 + 0.03125 \\
 &= 0.21875.
 \end{aligned}$$

$$\begin{aligned} P(\text{Joker appears twice}) &= P(J_1J_2) \\ &= 0.015625. \end{aligned}$$

$$\begin{aligned} \text{E.V.} &= (+3)(0.21875) + (+6)(0.015625) + (-1)(1 - 0.21875 - 0.015625) \\ &= -0.015625. \end{aligned}$$

This represents a 1.5625% house advantage on the Single Joker bet.

Double Joker Bet

$$\begin{aligned} P(\text{Joker appears twice}) &= P(J_1J_2) \\ &= 0.015625. \end{aligned}$$

$$\begin{aligned} \text{E.V.} &= (+60)(0.015625) + (-1)(1 - 0.015625) \\ &= -0.046875. \end{aligned}$$

This represents a 4.6875% house advantage on the Double Joker bet.

House Advantage – Overall Casino Win

The overall game advantage is a weighted linear average of the house edges for each bet type, with weights equal to the proportion of the handle for each bet. Since the house advantage varies for the different bets, the overall casino win will depend on the relative amounts wagered on these different bets. Under the assumption that equal amounts are bet on each of the twelve possible wagers, the overall casino edge is 4.56%. If the amounts wagered on each type of bet are not equal, the overall house advantage will be slightly higher or lower than 4.56%, depending on the betting pattern.

Note that the 12 possible bets can be classified into five groups according to house advantage:

- (1) Non-Yellow Specific Color, Single Joker (4 bets, H.A. = 1.5625%)
- (2) Yellow Specific Color (1 bet, H.A. = 6.25%)
- (3) Non-Yellow Color Combo (3 bets, H.A. = 7.03125%)
- (4) Yellow Color Combo (3 bets, H.A. = 5.46875%)
- (5) Double Joker (1 bet, H.A. = 4.6875%)

It is the precise mix of amounts wagered in these five categories that determines the overall casino win. Key factors are the bet type – Specific Color (including Single Joker), Color Combo, or Double Joker – and whether or not the bet involves Yellow. Table 4 in the Appendix displays the overall house edge for various betting pattern scenarios.

Although it is theoretically possible for the overall house advantage to be as low as 1.56250% or as high as 7.03125%, these are extreme cases each resulting only if all money wagered were on either (a) a combination of Non-Yellow Specific Colors and Single Joker bets, which would

yield a 1.56250% advantage, or (b) a combination of Non-Yellow Color Combo bets, yielding a 7.03125% advantage. It is reasonable to expect that wagers will be placed on bets in all categories with the resulting overall house edge somewhere between these two extremes.

It is possible to lump the twelve bets into two broad groups according to the relative size of the house advantage: Group 1, consisting of those bets with a relatively large house advantage and including the Color Combo (particularly Non-Yellow), Yellow Specific Color, and Double Joker bets, and Group 2, consisting of those with relatively low house advantage and including the Non-Yellow Specific Color and Single Joker bets. With these groups, generally the overall casino advantage will be greater when more money is wagered on those bets in Group 1 while the overall advantage will be lower when more money is wagered on those bets in Group 2.

Hit Frequencies

The *hit frequency* represents the percentage of times (bets) on average that a wager will result in a positive payoff to the player. The hit frequency is helpful in evaluating the behavior of a wager and is, for example, a standard figure reported on slot machine par sheets. For the Spin & Win game, the hit frequencies vary among the different bets and are given in Table 5. These figures can be interpreted as follows. A player betting on a Non-Yellow (i.e., Red, Green, or Blue) Specific Color will win something (either 0.5 to 1 or even money) about 61% of the time. Similarly, the Yellow Specific Color bet will pay the player (even money or 2 to 1) almost 44% of the time, Non-Yellow Combo bets will pay the player (2.5 to 1) 26.6% of the time, Yellow Color Combo bets will pay out (4.5 to 1) about 17% of the time, Single Joker bets will pay out (3 to 1 or 6 to 1) about 23.4% of the time, and the Double Joker bet will pay (60 to 1) about 1.6% of the time. Equivalently, one can say these six types of bets – Non-Yellow Specific Color, Yellow Specific Color, Non-Yellow Color Combo, Yellow Color Combo, Single Joker, and Double Joker – will result in a win to the player about once every 1.64, 2.29, 3.76, 5.82, 4.27, and 64.00 bets, respectively. Under the assumption that equal amounts are bet on each of the twelve possible wagers, the overall hit frequency is 31.9%, meaning players would experience a winning payoff almost one-third of the time.

APPENDIX

Spin & Win

Two 8-sided spinners: 2 Red, 2 Green, 2 Blue, 1 Yellow, 1 Joker

Table 1. Probability Distribution of Final Outcomes – Order Relevant					
Outcome #	1st Spin	2nd Spin	P(1st)	P(2nd)	P(Outcome)
1	Red	Red	0.250	0.250	0.062500
2	Red	Green	0.250	0.250	0.062500
3	Red	Blue	0.250	0.250	0.062500
4	Red	Yellow	0.250	0.125	0.031250
5	Red	Joker	0.250	0.125	0.031250
6	Green	Red	0.250	0.250	0.062500
7	Green	Green	0.250	0.250	0.062500
8	Green	Blue	0.250	0.250	0.062500
9	Green	Yellow	0.250	0.125	0.031250
10	Green	Joker	0.250	0.125	0.031250
11	Blue	Red	0.250	0.250	0.062500
12	Blue	Green	0.250	0.250	0.062500
13	Blue	Blue	0.250	0.250	0.062500
14	Blue	Yellow	0.250	0.125	0.031250
15	Blue	Joker	0.250	0.125	0.031250
16	Yellow	Red	0.125	0.250	0.031250
17	Yellow	Green	0.125	0.250	0.031250
18	Yellow	Blue	0.125	0.250	0.031250
19	Yellow	Yellow	0.125	0.125	0.015625
20	Yellow	Joker	0.125	0.125	0.015625
21	Joker	Red	0.125	0.250	0.031250
22	Joker	Green	0.125	0.250	0.031250
23	Joker	Blue	0.125	0.250	0.031250
24	Joker	Yellow	0.125	0.125	0.015625
25	Joker	Joker	0.125	0.125	0.015625
				Total	1.000000

Table 2. Probability Distribution of Final Outcomes – Order Ignored		
Outcome #	Outcome*	P(Outcome)
1	R ₁ R ₂	0.062500
2	G ₁ G ₂	0.062500
3	B ₁ B ₂	0.062500
4	Y ₁ Y ₂	0.015625
5	J ₁ J ₂	0.015625
6	R ₁ G ₂ or G ₁ R ₂	0.125000
7	R ₁ B ₂ or B ₁ R ₂	0.125000
8	G ₁ B ₂ or B ₁ G ₂	0.125000
9	Y ₁ R ₂ or R ₁ Y ₂	0.062500
10	Y ₁ G ₂ or G ₁ Y ₂	0.062500
11	Y ₁ B ₂ or B ₁ Y ₂	0.062500
12	J ₁ R ₂ or R ₁ J ₂	0.062500
13	J ₁ G ₂ or G ₁ J ₂	0.062500
14	J ₁ B ₂ or B ₁ J ₂	0.062500
15	J ₁ Y ₂ or Y ₁ J ₂	0.031250
	Total	1.000000
*R = Red, G = Green, B = Blue, Y = Yellow, J= Joker		

Table 3. House Advantages on Each Bet						
Bet	Payoff1	P(Payoff1)	Payoff2	P(Payoff2)	E.V.	H.A.
RED	0.5	0.468750	1.0	0.140625	-0.0156250	1.56250%
GREEN	0.5	0.468750	1.0	0.140625	-0.0156250	1.56250%
BLUE	0.5	0.468750	1.0	0.140625	-0.0156250	1.56250%
YELLOW	1.0	0.375000	2.0	0.062500	-0.0625000	6.25000%
RED-GREEN	2.5	0.265625	-	-	-0.0703125	7.03125%
RED-BLUE	2.5	0.265625	-	-	-0.0703125	7.03125%
GREEN-BLUE	2.5	0.265625	-	-	-0.0703125	7.03125%
YELLOW-RED	4.5	0.171875	-	-	-0.0546875	5.46875%
YELLOW-GREEN	4.5	0.171875	-	-	-0.0546875	5.46875%
YELLOW-BLUE	4.5	0.171875	-	-	-0.0546875	5.46875%
SINGLE JOKER	3.0	0.218750	6.0	0.015625	-0.0156250	1.56250%
DOUBLE JOKER	-	-	60.0	0.015625	-0.0468750	4.68750%

Table 4. Overall House Advantage

Table 4. Overall House Advantage										
	(2 bets)	(4 bets)	(6 bets)	R-G-B Specific Color	Yellow Specific Color	R-G-B Color Combo	Yellow Color Combo	1-Joker	2-Joker	Overall H.A.
	Jokers	%Specific	%Combo	1.56250%	6.25000%	7.03125%	5.46875%	1.56250%	4.68750%	
1	16.7%	16.7%	66.7%	0.125	0.042	0.333	0.333	0.083	0.083	5.1432%
2	16.7%	16.7%	66.7%	0.125	0.042	0.333	0.333	0.125	0.042	5.0130%
3	16.7%	16.7%	66.7%	0.125	0.042	0.333	0.333	0.042	0.125	5.2734%
4	16.7%	33.3%	50.0%	0.250	0.083	0.250	0.250	0.083	0.083	4.5573%
5	16.7%	33.3%	50.0%	0.250	0.083	0.250	0.250	0.125	0.042	4.4271%
6	16.7%	33.3%	50.0%	0.250	0.083	0.250	0.250	0.042	0.125	4.6875%
7	16.7%	41.7%	41.7%	0.313	0.104	0.208	0.208	0.083	0.083	4.2643%
8	16.7%	41.7%	41.7%	0.313	0.104	0.208	0.208	0.125	0.042	4.1341%
9	16.7%	41.7%	41.7%	0.313	0.104	0.208	0.208	0.042	0.125	4.3945%
10	16.7%	50.0%	33.3%	0.375	0.125	0.167	0.167	0.083	0.083	3.9714%
11	16.7%	50.0%	33.3%	0.375	0.125	0.167	0.167	0.125	0.042	3.8411%
12	16.7%	50.0%	33.3%	0.375	0.125	0.167	0.167	0.042	0.125	4.1016%
13	16.7%	66.7%	16.7%	0.500	0.167	0.083	0.083	0.083	0.083	3.3854%
14	16.7%	66.7%	16.7%	0.500	0.167	0.083	0.083	0.125	0.042	3.2552%
15	16.7%	66.7%	16.7%	0.500	0.167	0.083	0.083	0.042	0.125	3.5156%
16	50.0%	10.0%	40.0%	0.075	0.025	0.200	0.200	0.250	0.250	4.3359%
17	50.0%	10.0%	40.0%	0.075	0.025	0.200	0.200	0.375	0.125	3.9453%
18	50.0%	10.0%	40.0%	0.075	0.025	0.200	0.200	0.125	0.375	4.7266%
19	50.0%	20.0%	30.0%	0.150	0.050	0.150	0.150	0.250	0.250	3.9844%
20	50.0%	20.0%	30.0%	0.150	0.050	0.150	0.150	0.375	0.125	3.5938%
21	50.0%	20.0%	30.0%	0.150	0.050	0.150	0.150	0.125	0.375	4.3750%
22	50.0%	25.0%	25.0%	0.188	0.063	0.125	0.125	0.250	0.250	3.8086%
23	50.0%	25.0%	25.0%	0.188	0.063	0.125	0.125	0.375	0.125	3.4180%
24	50.0%	25.0%	25.0%	0.188	0.063	0.125	0.125	0.125	0.375	4.1992%
25	50.0%	30.0%	20.0%	0.225	0.075	0.100	0.100	0.250	0.250	3.6328%
26	50.0%	30.0%	20.0%	0.225	0.075	0.100	0.100	0.375	0.125	3.2422%
27	50.0%	30.0%	20.0%	0.225	0.075	0.100	0.100	0.125	0.375	4.0234%
28	50.0%	40.0%	10.0%	0.300	0.100	0.050	0.050	0.250	0.250	3.2813%
29	50.0%	40.0%	10.0%	0.300	0.100	0.050	0.050	0.375	0.125	2.8906%
30	50.0%	40.0%	10.0%	0.300	0.100	0.050	0.050	0.125	0.375	3.6719%
31	83.3%	3.3%	13.3%	0.025	0.008	0.067	0.067	0.417	0.417	3.5286%
32	83.3%	3.3%	13.3%	0.025	0.008	0.067	0.067	0.625	0.208	2.8776%
33	83.3%	3.3%	13.3%	0.025	0.008	0.067	0.067	0.208	0.625	4.1797%
34	83.3%	6.7%	10.0%	0.050	0.017	0.050	0.050	0.417	0.417	3.4115%
35	83.3%	6.7%	10.0%	0.050	0.017	0.050	0.050	0.625	0.208	2.7604%
36	83.3%	6.7%	10.0%	0.050	0.017	0.050	0.050	0.208	0.625	4.0625%
37	83.3%	8.3%	8.3%	0.063	0.021	0.042	0.042	0.417	0.417	3.3529%
38	83.3%	8.3%	8.3%	0.063	0.021	0.042	0.042	0.625	0.208	2.7018%
39	83.3%	8.3%	8.3%	0.063	0.021	0.042	0.042	0.208	0.625	4.0039%
40	83.3%	10.0%	6.7%	0.075	0.025	0.033	0.033	0.417	0.417	3.2943%
41	83.3%	10.0%	6.7%	0.075	0.025	0.033	0.033	0.625	0.208	2.6432%
42	83.3%	10.0%	6.7%	0.075	0.025	0.033	0.033	0.208	0.625	3.9453%
43	83.3%	13.3%	3.3%	0.100	0.033	0.017	0.017	0.417	0.417	3.1771%
44	83.3%	13.3%	3.3%	0.100	0.033	0.017	0.017	0.625	0.208	2.5260%
45	83.3%	13.3%	3.3%	0.100	0.033	0.017	0.017	0.208	0.625	3.8281%

Table 5. Hit Frequencies		
Wager Type	Hit Frequency	Once every ___ bets
Non-Yellow Specific Color	.609375	1.64
Yellow Specific Color	.437500	2.29
Non-Yellow Color Combo	.265625	3.76
Yellow Color Combo	.171875	5.82
Single Joker	.234375	4.27
Double Joker	.015625	64.00
Average if Equal Bet Distribution across the twelve individual bets	.319010	3.13